Why Pair Production Cannot Occur in a Vacuum (if it only involves a single neutral boson e.g. a photon)

A Simplified 'A'-Level Standard Explanation

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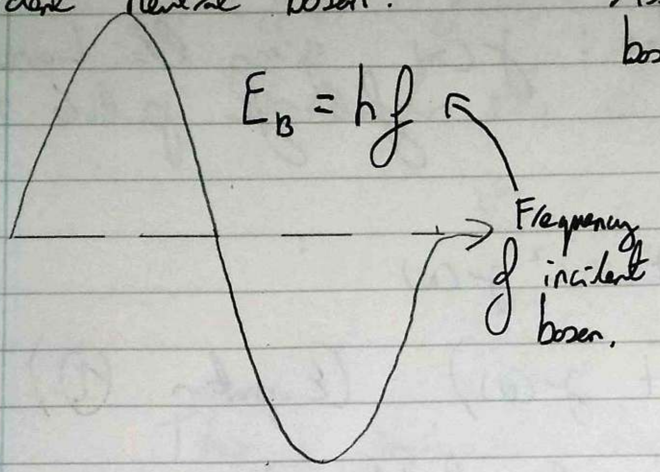
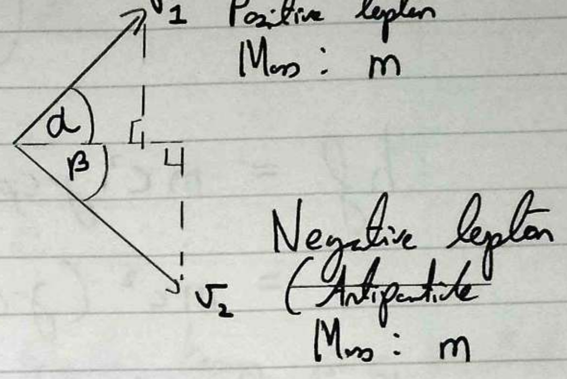
Based on the explanation presented at https://www.physicsforums.com/threads/pair-production-conservation-of-momentum-vs-conservation-of-energy.664025/ (Accessed 31st May 2017)

Why Pair Production Cannot occur in a Vacuum (if it only involves a single neutral boson (e.g. a photon)).

Incident neutral boson:



Assuming Pair Production for single neutral boson in vacuum:



The rest masses of the positive and negative leptons produced will be the same since one must be the anti-particle of the other.

By Principle of Conservation of Energy:

$$\mathcal{E}_B = E_p + E_N$$

(Where: $\mathcal{E}_B$ is the energy of the boson ~~(tot)~~ (Including energy due to rest mass.)
$\mathcal{E}_p$ — " — positive lepton
$\mathcal{E}_N$ — " — negative lepton)

$$\therefore hf = \mathcal{E}_{KE(P)} + \mathcal{E}_{M(P)} + \mathcal{E}_{KE(N)} + \mathcal{E}_{M(N)}$$

(Where: $\mathcal{E}_{KE(x)}$ shows kinetic energy of particle x.
$\mathcal{E}_{M(x)}$ shows mass energy of particle x.)

The total energy of an object is given by: $\mathcal{E} = \gamma mc^2$

Where γ is the Lorentz factor: $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

The kinetic energy alone will thus be the total energy minus the rest mass energy: $\mathcal{E}_{KE} = \gamma mc^2 - mc^2$

Hence:

$$hf = (mc^2\gamma(P) - mc^2) + mc^2 + (mc^2\gamma(N) - mc^2) + mc^2$$

(Where: $\gamma(x)$ gives the Lorentz factor for particle x.)

$$hf = mc^2\gamma(p) + mc^2\gamma(N)$$

$$= mc^2(\gamma(p) + \gamma(N)) \quad \text{(Equation 1)}$$

By Principle of Conservation of Momentum:

$$p_B = p_P + p_N$$

(Where: p_B is the momentum of the neutral boson.
p_P — " — positive lepton
p_N — " — negative lepton.)

For y direction:

$$0 = p_P \sin\alpha - p_N \sin\beta$$

$$0 = \gamma(P) m v_P \sin\alpha - \gamma(N) m v_N \sin\beta \quad (\text{Since: } p = \gamma m v)$$

For x direction:

Using the de Broglie hypothesis: $p = \frac{h}{\lambda}$:

$$\frac{h}{\lambda} = p_P \cos\alpha + p_N \cos\beta$$

Since: $\lambda = \frac{c}{f}$:

$$\frac{hf}{c} = \gamma(P) m v_P \cos\alpha + \gamma(N) m v_N \cos\beta$$

$$hf = mc(\gamma(P) v_P \cos\alpha + \gamma(N) v_N \cos\beta)$$

(Equation 2)

By equating equation 1 with equation 2:

$$mc^2(\gamma_{(p)} + \gamma_{(N)}) = mc(\gamma_{(p)} v_p \cos\alpha + \gamma_{(N)} v_N \cos\beta)$$

Hence for this to be true:

$$v_p \cos\alpha = c$$

And:

$$v_N \cos\alpha = c$$

This is impossible, hence single boson pair production cannot occur in a vacuum.